## Dynamic Programming and Applications Michael Schatz

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## Exact Matching Review

Where is GATTACA in the human genome?
$\mathrm{E}=183,105$
Brute Force
$(3 \mathrm{~GB})$
BANANA
BAN
ANA
NAN
ANA

Naive
Slow \& Easy

| Suffix Array (>15 GB) |  |
| :---: | :---: |
| 6 | \$ |
| 5 | A\$ |
| 3 | ANA\$ |
| 1 | ANANA\$ |
| 0 | BANANA\$ |
| 4 | NA\$ |
| 2 | NANA\$ |
| Vmatch, PacBio Aligner |  |
| Binary Search |  |

BWT/FM-Index
$(\sim 3 G B)$
\$BANANA
A\$BANAN
ANA\$BAN
ANANA\$B
BANANAS
NA\$BANA
NANA\$BA

Bowtie/BWA/SGA
Indexed Searching

These are general techniques useful for any search problem

## Agenda

I. Background on Dynamic Programming
I. Fibonacci Sequences
2. Longest-Increasing-Subsequences
2. Edit Distance \& Alignment
I. Computing Edit Distances
2. Global vs Local Alignment
3. Applications
I. Dynamic Time Warping
2. BLAST

## First:

A quick warm-up exercise

## Fibonacci Sequence



## Fibonacci Sequence

```
def fib(n):
if n == 0 or n == l:
        return n
    else:
    return fib(n-l) + fib(n-2)
```


[How long would it take for $F(7)$ ?]
[What is the running time?]

## Bottom-up Fibonacci Sequence

```
def fib(n):
    table = [0]* (n+I)
    table[0] = 0
    table[I] = I
    for i in range(2,n+I):
        table[i] = table[i-2] + table[i-1]
return table[n]
```

[How long will it take for $F(7)$ ?] [What is the running time?]


## Dynamic Programming

- General approach for solving (some) complex problems
- When applicable, the method takes far less time than naive methods.
- Polynomial time $\left(O(n)\right.$ or $O\left(n^{2}\right)$ instead of exponential time $\left(O\left(2^{n}\right)\right.$ or $\left.O\left(3^{n}\right)\right)$
- Requirements:
- Overlapping subproblems
- Optimal substructure
- Applications:
- Fibonacci
- Longest Increasing Subsequence
- Sequence alignment, Dynamic Time Warp,Viterbi
- Not applicable:
- Traveling salesman problem, Clique finding, Subgraph isomorphism, ...
- The cheapest flight from airport A to airport B involves a single connection through airport $C$, but the cheapest flight from airport $A$ to airport $C$ involves a connection through some other airport $D$.


## Second:

A quick interesting side problem

## Longest Increasing Subsequence

- Given a sequence of $N$ numbers $A_{1}, A_{2}, A_{3}, \ldots A_{N}$, find the longest monotonically increasing subsequence
- 29, 6, I4, 3I, 39, 78, 63, 50, I3, 64, 6I, I9
- Greedy approach (always extend the subsequence if you can):
$-\underline{29}, 6,14, \underline{31}, \underline{39}, \underline{78}, 63,50,13,64,61,19 \quad=>4$
- Brute force:
- Try all possible $O\left(2^{n}\right)$ subsequences

$$
\begin{array}{ll}
\underline{29}, 6,14,31,39,78,63,50,13,64,61,19 & =>~ I \\
\underline{29}, \underline{6}, 14,31,39,78,63,50,13,64,61,19 & =>\text { invalid } \\
\underline{29}, 6,14,31,39,78,63,50,13,64,61,19 & =>\text { invalid } \\
\underline{29}, 6,14, \underline{31}, 39,78,63,50,13,64,61,19 & =>2
\end{array}
$$

## Longest Increasing Subsequence

- Idea:
- The solution for all N numbers depends on the solution for the first $\mathrm{N}-\mathrm{I}$
- Look through the previous values to find the longest subsequence ending at $X$ such that $A_{X}<A_{N}$
- Dynamic Programming:
- Def: $L[i]$ is the longest increasing subsequence ending at position $j$
- Base case: $L[0]=0 \quad$ Recurrence: $L[j]=\max _{i<j}\{L[i]\}+1 \quad L I S=\max \{L[i]\}$

[What's the LIS of 0,8,4, I2,2, I0,6, I4, I,9,5, I3,3,II,7,I5 ?]


## Longest Increasing Subsequence

// Initialize

$$
\mathrm{L}[0]=0 ; \mathrm{P}[0]=0
$$

// Iteratively apply recurrence for $\mathrm{i}=\mathrm{I}$ to N
// find the best LIS to extend bestlis = 0; bestidx $=-\mathrm{I}$
for $\mathrm{j}=\mathrm{I}$ to i

if $((A[j]<=A[i])) \& \&(L[j]>$ bestlis $))$
bestlis $=\mathrm{L}[\mathrm{j}]$; bestid $\mathrm{x}=\mathrm{j}$
$\mathrm{L}[\mathrm{i}]=$ bestlis $+\mathrm{I} ; \mathrm{P}[\mathrm{i}]=$ bestidx
// Scan the L array to find the overall LIS

$$
\text { LIS }=0
$$

for $\mathrm{j}=\mathrm{I}$ to N
if (L[j] > LIS) LIS = L[i]
print "The LIS is \$LIS"
[What's the running time?]

And now for the main event!

## In-exact alignment

- Where is GATTACA approximately in the human genome?
- And how do we efficiently find them?
- It depends...
- Define 'approximately'
- Hamming Distance, Edit distance, or Sequence Similarity
- Ungapped vs Gapped vs Affine Gaps
- Global vs Local
- All positions or the single 'best'?
- Efficiency depends on the data characteristics \& goals
- Bowtie: BWT alignment for short read mapping
- Smith-Waterman: Exhaustive search for optimal alignments
- BLAST: Hash based homology searches
- MUMmer: Suffix Tree based whole genome alignment


## Similarity metrics

- Hamming distance
- Count the number of substitutions to transform one string into another

| GATTACA | ATTACCC |
| :---: | :---: |
| $\|\|\|\mathrm{X}\|\|\|$ | $\mathrm{XX}\|\mathrm{XX}\| \mathrm{X}$ |
| GATCACA | GATTACA |
| 1 | 5 |

- Edit distance
- The minimum number of substitutions, insertions, or deletions to transform one string into another

| GATTACA | -ATTACCC |
| :---: | :---: |
| $\|\|\|\mathrm{X}\|\|\|$ | $\mathrm{X}\|\|\|\|\mid \mathrm{XX}$ |
| GATCACA | GATTAC-A |
| 1 | 3 |

## Edit Distance Example

AGCACACA $\rightarrow$ ACACACTA in 4 steps

AGCACACA $\rightarrow$ (I. change $G$ to $C)$<br>ACCACACA $\rightarrow$ (2. delete C)<br>ACACACA $\rightarrow$ (3. change $A$ to $T$ )<br>ACACACT $\rightarrow$ (4. insert $A$ after $T$ )<br>ACACACTA $\rightarrow$ done

[Is this the best we can do?]

## Edit Distance Example

AGCACACA $\rightarrow$ ACACACTA in 3 steps

AGCACACA $\rightarrow$ (I. change $G$ to $C)$<br>ACCACACA $\rightarrow$ (2. delete C)<br>ACACACA $\rightarrow$ (3. insert $T$ after $3^{\text {rd }} \mathrm{C}$ )<br>ACACACTA $\rightarrow$ done

[Is this the best we can do?]

## Reverse Engineering Edit Distance $\mathrm{D}(\mathrm{AGCACACA}, ~ A C A C A C T A)=$ ?

Imagine we already have the optimal alignment of the strings, the last column can only be I of 3 options:

| ...M | ...I | ...D |
| :--- | :--- | :--- |
| .... | ... | ... |
|  | ...A |  |

The optimal alignment of last two columns is then I of 9 possibilities

| ...MM | ..IM | ..DM | ...MI | ...II | ...DI | ...MD | ...ID |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | ...DD

The optimal alignment of the last three columns is then I of 27 possibilities...

| ...M... | ...I... | $\ldots$. ... |
| :--- | :--- | :--- |
| ...X... | ..-... | $\ldots$..... |
| ...Y... | ...Y... | $\ldots-. .$. |

Eventually spell out every possible sequence of $\{I, M, D\}$

## Recursive solution

- Computation of $D$ is a recursive process.
- At each step, we only allow matches, substitutions, and indels
$-D(i, j)$ in terms of $D\left(i^{\prime}, j^{\prime}\right)$ for $i^{\prime} \leq i$ and $j^{\prime} \leq j$.
$D(A G C A C A C A, A C A C A C T A)=\min \{D(A G C A C A C A, ~ А С А С А С Т)+I$, D(AGCACAC, АСАСАСТА) +1 , $D($ AGCACAC, ACACACT) $+\delta(A, A)\}$

[What is the running time?]


## Dynamic Programming

- We could code this as a recursive function call... ...with an exponential number of function evaluations
- There are only $(\mathrm{n}+\mathrm{l}) \times(\mathrm{m}+\mathrm{l})$ pairs i and j
- We are evaluating $D(i, j)$ multiple times
- Compute D(i,j) bottom up.
- Start with smallest $(\mathrm{i}, \mathrm{j})=(\mathrm{I}, \mathrm{I})$.
- Store the intermediate results in a table.
- Compute $\mathrm{D}(\mathrm{i}, \mathrm{j})$ after $\mathrm{D}(\mathrm{i}-\mathrm{I}, \mathrm{j}), \mathrm{D}(\mathrm{i}, \mathrm{j}-\mathrm{I})$, and $\mathrm{D}(\mathrm{i}-\mathrm{I}, \mathrm{j}-\mathrm{I})$


## Recurrence Relation for $D$

Find the edit distance (minimum number of operations to convert one string into another) in $\mathrm{O}(\mathrm{mn})$ time

- Base conditions:

$$
\begin{aligned}
& -D(i, 0)=i, \text { for all } i=0, \ldots, n \\
& -D(0, j)=j, \text { for all } j=0, \ldots, m
\end{aligned}
$$

- For $\mathrm{i}>0, \mathrm{j}>0$ :

$$
D(i, j)=\min \{
$$

$$
\begin{array}{ll}
D(i-I, j)+I, & / / \text { align } 0 \text { chars from } S, I \text { from } T \\
D(i, j-I)+I, & / / \text { align I chars from } S, 0 \text { from } T \\
D(i-I, j-I)+\delta(S(i), T(j)) / / \text { align } I+I \text { chars }
\end{array}
$$

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{A}$ | 1 |  |  |  |  |  |  |  |  |
| $\mathbf{G}$ | 2 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 3 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 4 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 5 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 7 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 8 |  |  |  |  |  |  |  |  |

[What does the initialization mean?]

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{A}$ | 1 | 0 |  |  |  |  |  |  |  |
| $\mathbf{G}$ | 2 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 3 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 4 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 5 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 7 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 8 |  |  |  |  |  |  |  |  |

$$
\mathrm{D}[\mathrm{~A}, \mathrm{~A}]=\min \{\mathrm{D}[\mathrm{~A},]+1, \mathrm{D}[, \mathrm{~A}]+1, \mathrm{D}[,]+\delta(\mathrm{A}, \mathrm{~A})\}
$$

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | $\mathbf{1}$ | $\mathbf{2}$ | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{A}$ | $\mathbf{1}$ | 0 | $\mathbf{1}$ |  |  |  |  |  |  |
| $\mathbf{G}$ | 2 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 3 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 4 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 5 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 7 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 8 |  |  |  |  |  |  |  |  |

$$
\mathrm{D}[\mathrm{~A}, \mathrm{AC}]=\min \{\mathrm{D}[\mathrm{~A}, \mathrm{~A}]+1, \mathrm{D}[, \mathrm{AC}]+1, \mathrm{D}[, \mathrm{~A}]+\delta(\mathrm{A}, \mathrm{C})\}
$$

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{A}$ | 1 | 0 | 1 | 2 |  |  |  |  |  |
| $\mathbf{G}$ | 2 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 3 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 4 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 5 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 7 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 8 |  |  |  |  |  |  |  |  |

$D[A, A C A]=\min \{D[A, A C]+1, D[, A C A]+1, D[, A C]+\delta(A, A)\}$

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underline{0}$ | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ | $\underline{4}$ | $\underline{5}$ | $\underline{6}$ | $\underline{7}$ | 8 |
| $\mathbf{A}$ | 1 | 0 | $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | $\underline{7}$ |
| $\mathbf{G}$ | 2 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 3 |  |  |  |  |  |  |  |  |
| A | 4 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 5 |  |  |  |  |  |  |  |  |
| A | 6 |  |  |  |  |  |  |  |  |
| $\mathbf{C}$ | 7 |  |  |  |  |  |  |  |  |
| $\mathbf{A}$ | 8 |  |  |  |  |  |  |  |  |

$D[A, A C A C A C T A]=7$

[What about the other A?] ACACACTA

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | C | A | C | A | C | $\mathbf{T}$ | A |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underline{0}$ | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ | $\underline{4}$ | 5 | 6 | 7 | 8 |
| A | $\mathbf{1}$ | 0 | 1 | 2 | 3 | $\underline{4}$ | 5 | 6 | 7 |
| $\mathbf{G}$ | 2 | 1 | 1 | 2 | 3 | 4 | $\underline{5}$ | $\underline{6}$ | $\underline{7}$ |
| C | 3 |  |  |  |  |  |  |  |  |
| A | 4 |  |  |  |  |  |  |  |  |
| C | 5 |  |  |  |  |  |  |  |  |
| A | 6 |  |  |  |  |  |  |  |  |
| C | 7 |  |  |  |  |  |  |  |  |
| A | 8 |  |  |  |  |  |  |  |  |

$D[A G, A C A C A C T A]=7$

$$
\begin{aligned}
& ----A G-- \\
& * * * * \mid * * * \\
& \text { АСАСАСТА }
\end{aligned}
$$

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underline{0}$ | $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{A}$ | 1 | $\underline{0}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{G}$ | 2 | $\underline{1}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{C}$ | 3 | 2 | $\underline{1}$ | 2 | 2 | 3 | 4 | 5 | 6 |
| $\mathbf{A}$ | 4 | 3 | 2 | $\underline{1}$ | 2 | 2 | 3 | 4 | 5 |
| $\mathbf{C}$ | 5 | 4 | 3 | 2 | $\underline{1}$ | 2 | 2 | 3 | 4 |
| $\mathbf{A}$ | 6 | 5 | 4 | 3 | 2 | $\underline{1}$ | 2 | 3 | 3 |
| $\mathbf{C}$ | 7 | 6 | 5 | 4 | 3 | 2 | $\underline{1}$ | $\underline{2}$ | 3 |
| $\mathbf{A}$ | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | $\underline{2}$ |

D[AGCACACA,ACACACTA $]=2$

$$
\begin{aligned}
& \text { AGCACAC-A } \\
& |*|||||*| \\
& \text { A-САСАСТА }
\end{aligned}
$$

[Can we do it any better?]

Break

## Dynamic Programming Matrix

|  |  | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{A}$ | $\mathbf{C}$ | $\mathbf{T}$ | $\mathbf{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underline{0}$ | $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{A}$ | 1 | $\underline{0}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{G}$ | 2 | $\underline{1}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{C}$ | 3 | 2 | $\underline{1}$ | 2 | 2 | 3 | 4 | 5 | 6 |
| $\mathbf{A}$ | 4 | 3 | 2 | $\underline{1}$ | 2 | 2 | 3 | 4 | 5 |
| $\mathbf{C}$ | 5 | 4 | 3 | 2 | $\underline{1}$ | 2 | 2 | 3 | 4 |
| $\mathbf{A}$ | 6 | 5 | 4 | 3 | 2 | $\underline{1}$ | 2 | 3 | 3 |
| $\mathbf{C}$ | 7 | 6 | 5 | 4 | 3 | 2 | $\underline{1}$ | $\underline{2}$ | 3 |
| $\mathbf{A}$ | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | $\underline{2}$ |

D[AGCACACA,ACACACTA $]=2$

$$
\begin{aligned}
& \text { AGCACAC-A } \\
& |*|||||*| \\
& \text { A-САСАСТА }
\end{aligned}
$$

[Can we do it any better?]

## Global Alignment Schematic



A high quality alignment will stay close to the diagonal

- If we are only interested in high quality alignments, we can skip filling in cells that can't possibly lead to a high quality alignment
- Find the global alignment with at most edit distance d: $\mathrm{O}(2 \mathrm{dn})$

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## Sequence Similarity

- Similarity score generalizes edit distance
- Certain mutations are much more likely than others
- Hydrophilic -> Hydrophillic much more likely than Hydrophillic -> Hydrophobic
- BLOSSUM62
- Empirically measure substitution rates among proteins that are $62 \%$ identical
- Positive score: more likely than chance, Negative score: less likely



## Edit Distance and Global Similarity

$$
\begin{aligned}
D(i, j)=\min \{ & \\
& D(i-I, j)+I, \\
& D(i, j-I)+I, \\
& \quad D(i-I, j-I)+\delta(S(i), T(j))
\end{aligned}
$$

$s=4 \times 4$ or $20 \times 20$ scoring matrix

$$
\begin{aligned}
S(i, j)=\max \{ & \\
& S(i-I, j)-I, \\
& S(i, j-I)-I, \\
& S(i-I, j-I)+s(S(i), T(j))
\end{aligned}
$$

## Local vs. Global Alignment

- The Global Alignment Problem tries to find the best end-to-end alignment between the two strings
- Only applicable for very closely related sequences
- The Local Alignment Problem tries to find pairs of substrings with highest similarity.
- Especially important if one string is substantially longer than the other
- Especially important if there is only a distant evolutionary relationship


## Global vs Local Alignment Schematic



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## Local vs. Global Alignment (cont d)

- Global Alignment

- Local Alignment-better alignment to find conserved segment
tccCAGTTATGTCAGgggacacgagcatgcagagac |||||||||||
aattgccgccgtcgttttcagCAGTTATGTCAGatc


## The Local Alignment Recurrence

- The largest value of $s_{i, j}$ over the whole edit graph is the score of the best local alignment.
- The recurrence:

$$
s_{i, j}=\max \left\{\begin{array}{l}
0 \\
s_{i-1, j-1}+\delta\left(v_{i}, w_{j}\right) \\
s_{i-1, j}+\delta\left(v_{i},-\right) \\
s_{i, j-1}+\delta\left(-, w_{j}\right)
\end{array}\right.
$$

Power of ZERO: there is only this change from the original recurrence of a Global Alignment - since there is only one "free ride" edge entering into every vertex

## G-Local Alignments: Searching for GATTACA



- Don' t "charge" for optimal alignment starting in cells $(0, j)$

Base conds: $D(0, j)=0, D(i, 0)=\Sigma_{k \leq i} s\left(S(k),{ }^{-}-{ }^{\prime}\right)$
Don' t "charge" for ending alignment at end of $P$ (but not necc.T)

- Find cell ( $\mathrm{n}, \mathrm{j}$ ) with edit distance $\leq \delta$

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## Affine Gap Penalties

- In nature, a series of $k$ indels often come as a single event rather than a series of $k$ single nucleotide events:

ATA__GC
ATATTGC

Normal scoring would
This is more likely.
give the same score for both alignments

ATAG_GC
AT_GTGC

This is less
likely.

## Accounting for Gaps

- Gaps- contiguous sequence of spaces in one of the rows
- Score for a gap of length $x$ is: $-(\rho+\sigma x)$ where $\rho>0$ is the gap opening penalty
$\rho$ will be large relative to gap extension penalty $\sigma$
- Gap of length $\mathrm{I}:-(\rho+\sigma)=-6$
- Gap of length 2: $-(\rho+\sigma 2)=-7$
- Gap of length 3: $-(\rho+\sigma 3)=-8$
- Smith-Waterman-Gotoh incorporates affine gap penalties without increasing the running time $\mathrm{O}(\mathrm{mn})$

Break

## Dynamic Time Warp

- Algorithm for measuring the similarity between two sequences of numeric values that vary in time or speed
- Computes a non-linear mapping for sequence $A$ to sequence $B$
- Many applications for video, audio, and graphics
- Speech processing: Recognize speech patterns coping with different speaking speeds
- EEG processing: Identify anomalies in brain or heart activity



## DTW Algorithm

- DP Algorithm
- Input: two time series C and Q
- Compute the time warping matrix $d$

$$
\begin{aligned}
& d(0,0)=0 ; d(i, 0)=d(0, j)=\infty \\
& d(i, j)=\left|c_{i}-q_{j}\right|+\min \left\{\begin{array}{l}
d(i-I, j) \\
d(i, j-I) \\
d(i-I, j-I)
\end{array}\right.
\end{aligned}
$$

- Warping matrix projects sequence to sequence $Q$, allowing for non-
 linear contractions and expansions.



## Basic L_ocal Alignment Search Tool

- Rapidly compare a sequence Q to a database to find all sequences in the database with an score above some cutoff $S$.
- Which protein is most similar to a newly sequenced one?
- Where does this sequence of DNA originate?
- Speed achieved by using a procedure that typically finds "most" matches with scores > S.
- Tradeoff between sensitivity and specificity/speed
- Sensitivity - ability to find all related sequences
- Specificity - ability to reject unrelated sequences


## Seed and Extend

```
FAKDFLAGGVAAAISKTAVAPIERVKLLLQVQHASKQITADKQYKGIIDCVVRIPKEQGV
F D +GG AAA+SKTAVAPIERVKLLLQVQ ASK I DK+YKGI+D ++R+PKEQGV
FLIDLASGGTAAAVSKTAVAPIERVKLLLQVQDASKAIAVDKRYKGIMDVLIRVPKEQGV
```

- Homologous sequence are likely to contain a short high scoring word pair, a seed.
- Unlike Baeza-Yates, BLAST *doesn't* make explicit guarantees
- BLAST then tries to extend high scoring word pairs to compute maximal high scoring segment pairs (HSPs).
- Heuristic algorithm but evaluates the result statistically.


## BLAST - Algorithm -

- Step I: Preprocess Query

Compile the short-high scoring word list from query. The length of query word, $w$, is 3 for protein scoring Threshold T is 13


## BLAST - Algorithm -

- Step 2: Construct Query Word Hash Table

Query: LAALLNKCKTPQGQRLVNQWIKQPLMD


## BLAST - Algorithm -

- Step 3: Scanning DB

Identify all exact matches with DB sequences


## BLAST - Algorithm -

- Step 4 (Search optimal alignment) For each hit-word, extend ungapped alignments in both directions.
Let $S$ be a score of hit-word
- Step 5 (Evaluate the alignment statistically)

Stop extension when E-value (depending on score S) become less than threshold. The extended match is called High Scoring Segment Pair.

E-value $=$ the number of HSPs having score S (or higher) expected to occur by chance.
$\rightarrow$ Smaller E-value, more significant in statistics
Bigger E-value, by chance
$E[\#$ occurrences of a string of length $m$ in reference of length $L] \sim L / 4^{m}$

## BLAST E-values

The expected number of HSPs with the score at least $S$ is :

$$
\mathrm{E}=\mathrm{K}^{*} \mathrm{n}^{*} \mathrm{~m}^{*} \mathrm{e}^{-\lambda \mathrm{s}}
$$

$\mathrm{K}, \lambda$ are constant depending on model $\mathrm{n}, \mathrm{m}$ are the length of query and sequence

The probability of finding at least one such HSP is:
$P=I-e^{E}$
$\rightarrow$ If a word is hit by chance ( E -value is bigger), P become smaller.

The distribution of Smith-Waterman local alignment scores between two random sequences follows the Gumbel extreme value distribution

## Very Similar Sequences

Query: HBA_HUMAN Hemoglobin alpha subunit
Sbjct: HBB_HUMAN Hemoglobin beta subunit
Score $=114$ bits (285), Expect $=1 e-26$
Identities $=61 / 145$ (42\%), Positives $=86 / 145$ (59\%), Gaps $=8 / 145$ (5\%)
Query 2 LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF------DLSHGSAQV 55 
Sbjct 3 LTPEEKSAVTALWGKV--NVDEVGGEALGRLLVVYPWTQRFFESFGDLSTPDAVMGNPKV ..... 60
Query 56 KGHGKKVADALTNAVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHLPA 115K HGKKV A ++ +AH+D++ + LS $+L H \quad K L$ VDP NF+LL $+L+L A H$
Sbjct 61 KAHGKKVLGAFSDGLAHLDNLKGTFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHFGK ..... 120
Query 116 EFTPAVHASLDKFLASVSTVLTSKY ..... 140
EFTP V A+ K +A V+ L KY
Sbjct 121 EFTPPVQAAYQKVVAGVANALAHKY ..... 145

## Quite Similar Sequences

Query: HBA_HUMAN Hemoglobin alpha subunitSbjct: MYG_HUMAN Myoglobin
Score $=51.2$ bits (121), Expect $=1 e-07$,
Identities $=38 / 146$ (26\%) , Positives $=58 / 146$ (39\%) , Gaps $=6 / 146$ (4\%)
Query 2 LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF------DLSHGSAQV ..... 55 
Sbjct 3 LSDGEWQLVLNVWGKVEADIPGHGQEVLIRLFKGHPETLEKFDKFKHLKSEDEMKASEDI ..... 62
Query 56 KGHGKKVADALTNAVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHLPA ..... 115
$\mathrm{K} \mathrm{HG} \mathrm{V} \mathrm{AL}+\quad+\mathrm{L}+\mathrm{HA} \mathrm{K} \mathrm{++}$ + +S C++ L +P
Sbjct 63 KKHGATVLTALGGILKKKGHHEAEIKPLAQSHATKHKIPVKYLEFISECIIQVLQSKHPG ..... 122
Query 116 EFTPAVHASLDKFLASVSTVITSKYR ..... 141
$+F \quad+++K$ L $+S Y+$
Sbjct 123 DFGADAQGAMNKALELFRKDMASNYK ..... 148

## Not similar sequences

```
Query: HBA_HUMAN Hemoglobin alpha subunit
Sbjct: SPAC869.02c [Schizosaccharomyces pombe]
    Score = 33.1 bits (74), Expect = 0.24
    Identities = 27/95 (28%), Positives = 50/95 (52%), Gaps = 10/95 (10%)
Query 30 ERMFLSFPTTKTYFPHFDLSHGSAQVKGHGKKVADALTNAVAHVDDMPNALSALSDLHAH }8
    ++M ++P P+F+ +H + + +A AL N ++DD+ +LSA D
Sbjct 59 QKMLGNYPEV---LPYFNKAHQISL--SQPRILAFALLNYAKNIDDL-TSLSAFMDQIVV 112
Query 90 K---LRVDPVNFKLLSHCLLVTLAAHLPAEF-TPA 120
    K L++ ++ ++ HCLL T+ LP++ TPA
Sbjct 113 KHVGLQIKAEHYPIVGHCLLSTMQELLPSDVATPA 147
```


## Blast Versions

| Program | Database | Query |
| :---: | :---: | :---: |
| BLASTN | Nucleotide | Nucleotide |
| BLASTP | Protein | Protein |
| BLASTX | Protein | Nucleotide translated in <br> to protein |
| TBLASTN | Nucleotide translated in <br> to protein | Protein |
| TBLASTX | Nucleotide translated in <br> to protein | Nucleotide translated in <br> to protein |

## NCBI Blast



- Nucleotide Databases
- nr:All Genbank
- refseq: Reference organisms
- wgs:All reads
- Protein Databases
- nr:All non-redundant sequences
- Refseq: Reference proteins


## BLAST Exercise

$>$ whoami
TTGATGCAGGTATCTGCGACTGAGACAATATGCA ACAGTTGAATGAATCATAATGGAATGTGCACTCT AACCAGCCAATTTGATGCTGGCTGCAGAGATGC AAGATCAAGAGGTGACACCTGCTCTGAAGAAAG CACAGTTGAACTGCTGGATCTGCAACTACAGCA GGCACTCCAGGCACCAAGACAACATCTTTTACA CCAGCAAACATGTGGATTGATATCTCCTAACAGC AGTGATTAACAGAGACGACTGCAGGATTTGCTTC CACAAACAAAAT

## Parameters

- Larger values of $w$ increases the number of neighborhood words, but decreases the number of chance matches in the database.
- Increasing w decreases sensitivity.
- Larger values of T decrease the overall execution time, but increase the chance of missing a MSP having score $\geq$ S.
- Increases T decreases the sensitivity
- Larger values of S increase the specificity. The value of $S$ is affected by changes in the expectation value parameter.


## Sequence Alignment Summary

- Distance metrics:
- Hamming: How many substitutions?
- Edit Distance: How many substitutions or indels?
- Sequence Similarity: How similar (under this model of similarity)?
- Techniques
- Seed-and-extend:Anchor the search for in-exact using exact only
- Dynamic Programming: Find a global optimal as a function of its parts
- BWT Search: implicit DFS of SA/ST
- Sequence Alignment Algorithms: Pick the right tool for the job
- Smith-Waterman: DP Local sequence alignment
- BLAST: Homology Searching
- Bowtie/BWA/Novoalign: short read mapping

